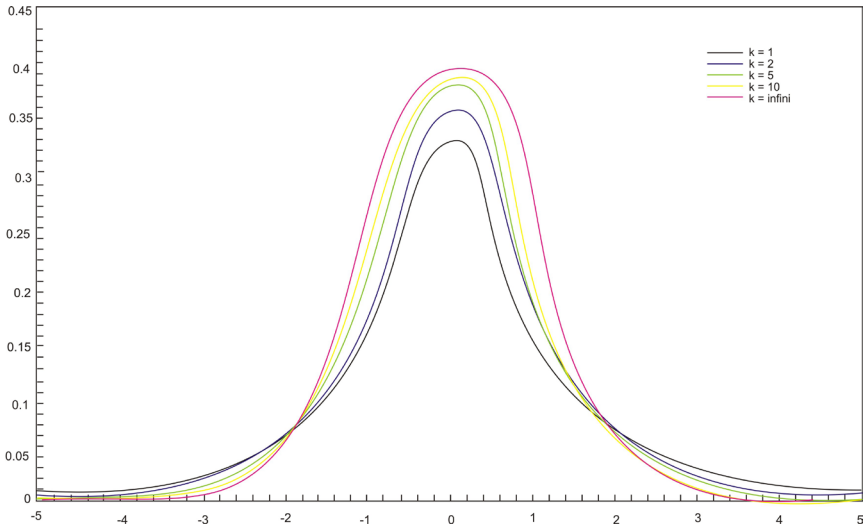
t-Distribution

Big Picture

The Central Limit Theorem allows us to approximate the sample distribution as a normal distribution whenever the sample size is greater than 30. It is not always possible, however, to have a large sample size due to costs, available time, or other limitations. When this is the case, we use the *t*-distribution (also known as Student's *t*-distribution).

Characteristics

- Student's *t*-distributions are a *family* of distributions.
- They are symmetrical, bell-shaped, and centered on a mean (just like normal distribution).
- The distribution shape changes as the sample size changes.
- Each distribution has a specific shape (distribution) depending on the number of degrees of freedom.
  - Number of degrees of freedom $k = n - 1$ (n is the sample size)
- The *t*-distribution looks more like the normal distribution as the sample size increases.
  - Once the sample size is large enough, around 120, then the normal distribution can be used instead of the *t*-distribution.



t-distributions have these characteristics:

- The mean of the distribution equals zero.
- The population standard deviation is unknown.
- Smaller sample sizes produce flatter curves.
- The population is unimodal and symmetric.

t-Test Statistic

To calculate the *t*-test statistic, use the formula: $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

- t is the test statistic and has n - 1 degrees of freedom
- $\bar{x}$ is the sample mean
- μ is the population mean
- s is the sample standard deviation
- n is the sample size
- $\frac{s}{\sqrt{n}}$ is the estimated standard error

It looks very similar to the formula for calculating the *z*-score, except σ is replaced with s.

Graphing Calculator

In a graphing calculator, we can use a command to find the t-distribution. The command is: tpdf(x, df). x is the t value, and df is the degrees of freedom. This will give us the area under the t curve.

There is another similar equation called tcdf, which requires us to plug in two values for x: one low and one high. This will give us the area under the t curve between those two values.

If you can't find the commands, check the manual for your graphing calculator. For the TI-83/TI-84, the commands can be found under [2ND][DISTR].

Notes